Econ 802

Lecture Notes for Chapter 18

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These notes are based on Ch. 18 in Varian.
How can we extend the definition of WE to the case where there are firms?

Two main issues: (1) we need a more complex way of describing market-clearing conditions, to include supply and demand by firms

(2) we need more complex budget constraints for consumers, to take account of firm profits

Notation: consumers $i = 1..n$
firms $j = 1..m$
goods $g = 1..k$

First, think about firms. Each firm j takes the price vector $p = (p_1 .. p_k) \geq 0$ as given; it has a production possibilities set Y_j, and chooses a production plan $y_j \in Y_j$ to solve

$$\max\ p y_j \quad \text{subject to } y_j \in Y_j$$

Assume this problem has a unique solution.

Write the firm's net supply function as $y_j(p)$

This is a k-dimensional vector (k goods)

Recall that if $y_{jg}(p) > 0$ then good g is a net output for firm j; if $y_{jg}(p) < 0$, then good g is a net input.

This yields the profit for firm j: $\pi_j(p) = p y_j(p)$

Next, think about consumers.

Each consumer i takes p as given; has a utility function $u_i(x_i)$; and chooses a bundle x_i to solve

$$\max\ u_i(x_i) \text{ subject to } p x_i \le p w_i + \sum_{j=1}^{m} T_{ij}\, p y_j(p)$$

where w_i is the endowment vector as before; T_{ij} is person i's share in the profit $p y_j(p)$ for firm j.

We assume $T_{ij} \geq 0$ for all i, j with

$$\sum_{i=1}^{n} T_{ij} = 1 \text{ for all } j.$$

This says that each firm's total profit must be distributed to consumers somehow. Note that some consumers may have zero ownership shares in some or all firms.

Also note: consumers take p, w_i, T_{ij}, and $y_j(p)$ as given (all are exogenous for consumer i)

In particular, T_{ij} is part of i's endowment.

However, if someone asked consumer i what firm j should do, the answer is: maximize profit! (everyone with $T_{ij} > 0$ for firm j agrees on this) The reason is simple: if a firm maximizes profit, the consumer has more income to spend.

Two things lead to this conclusion:

(a) perfect competition $\Rightarrow$ the decisions of one firm can't affect the prices facing any other firm

(b) no externalities $\Rightarrow$ one firm's production plan can't affect the production possibilities set of any other firm (or utility for any consumer)

Utility max by i gives the demand function $x_i(p)$

Again we are assuming a unique solution and again this is a k-dimensional vector.

Note: $x_i(p)$ is an abbreviation for the full Marshallian demand of the form $x_i[p, m_i(p)]$ where $m_i(p)$ comes from the right hand side of the budget constraint.

Finally, think about market clearing.

If we want no excess demand for any good in equilibrium (but excess supply is OK), then a Walrasian equilibrium price vector p^* must give

$$\sum_{i=1}^{n} x_i(p^*) \le \sum_{j=1}^{m} y_j(p^*) + \sum_{i=1}^{n} w_i$$

In general we have $x_i(p) \geq 0$ for all i.
But any individual $y_{jg}(p)$ could be >0 or <0;
some firms might produce good g as an output while others use it as an input, etc.
So net supply $\sum_{j=1}^{m} y_{jg}(p)$ for good g from the firms as a group might be either >0 or <0.
If this is >0, not a problem. If <0, then we must have $\sum_{i=1}^{n} w_{ig} > 0$ so the firms can get inputs from the endowments of the consumers (such as labor).
Varian explains this in more detail.

Now let's define the aggregate excess demand function:

$$z(p) \equiv \underbrace{\sum_{i=1}^{n} x_i(p)}_{\text{demand}} - \underbrace{\sum_{j=1}^{m} y_j(p) - \sum_{i=1}^{n} w_i}_{\text{supply (from both firms and endowments)}}$$

Note: everything here is a k-dimensional vector of goods

① continuity. As long as $x_i(p)$ and $y_j(p)$ are all continuous, $z(p)$ is continuous

② homogeneity of degree zero.
What happens to $x_i(p)$ if we multiply all prices by $t>0$?
In particular, what is the effect on income $m_i(p)$?

$$m_i(p) = pw_i + \sum_{j=1}^{M} T_{ij}\, py_j(p)$$

(a) we have $y_j(tp) = y_j(p)$ due to zero homogeneity of net supply (multiplying all prices by t doesn't change the optimal production plan for firm j)

(b) so $m_i(tp) = t m_i(p)$

(c) so $x_i[tp, m_i(tp)] = x_i[tp, tm_i(p)]$

$$= x_i[p, m_i(p)]$$

because Marshallian demands are homogeneous of degree zero in (p, m)

Therefore $z(p)$ is homogeneous of degree zero.

3 Walras's Law

Is it true that $pz(p) \equiv 0$ for all p?

Use the definition of $z(p)$:

$$pz(p) = p\sum_i x_i(p) - p\sum_j y_j(p) - p\sum_i w_i$$

$$= \sum_i \underbrace{[px_i(p) - pw_i]} - \sum_j py_j(p)$$

if the budget constraints hold with equality, we have

$$px_i(p) = pw_i + \sum_j T_{ij}\, py_j(p)$$

So we get $pz(p) = \sum_i \sum_j T_{ij}\, py_j(p) - \sum_j py_j(p)$

$$\Rightarrow pz(p) = \sum_j py_j(p) \underbrace{\left(\sum_i T_{ij}\right)}_{=1 \text{ for all } j} - \sum_j py_j(p)$$

Therefore we get $pz(p) \equiv 0$ for all p,
which is what we wanted to show;
Walras's Law still holds with firms and production.

Now what about the existence of some p^*
such that $z(p^*) \leq 0$?

Yes, such a p^* exists. The proof is exactly the
same as on p. 321 in Ch. 17 for the pure
exchange economy.
As long as $z(p)$ satisfies continuity, homogeneity
of degree zero, and Walras's Law, there is
some $p^* \geq 0$ such that $z(p^*) \leq 0$
where $z_g(p^*) < 0$ implies $p_g^* = 0$
so if a good is in excess supply, it has a zero price.

And as before, if every good is <u>desirable</u>, ie
$p_g = 0$ implies $z_g(p) > 0$,
then $z(p^*) = 0$ must hold and all markets
must clear exactly (supply = demand for all goods).

What about the welfare theorems?

Define an allocation to be (x, y) where
$x = (x_1 \ldots x_n)$ is a list of all consumption bundles
$y = (y_1 \ldots y_m)$ is a list of all production plans.

First Theorem: if ~~p,x~~ (p, x, y) is a WE
then (x, y) is PE.

Proof: Suppose (x, y) is not PE. Then there is some feasible (x', y') such that $x_i' \succ_i x_i$ for all i (assuming strong monotonicity here)

Since x_i maxes utility at prices p, it must be true that

$$p x_i' > p x_i = p w_i + \sum_j T_{ij} p y_j \quad \text{for } i = 1 \ldots n$$

Sum over i: $p \sum_i x_i' > \sum_i p w_i + \sum_j p y_j$

(using $\sum_i T_{ij} = 1$ for all j)

The feasibility of (x', y') implies

$$\sum_i x_i' = \sum_j y_j' + \sum_i w_i$$

(ignore $<$ here; if $<$ then just hand out more of the relevant goods and make x_i' even bigger for all i)

Substitute for $\sum_i x_i'$ in the previous inequality to get

$$p\left[\sum_j y_j' + \sum_i w_i\right] > \sum_i p w_i + \sum_j p y_j$$

This implies $\sum_j p y_j' > \sum_j p y_j$ where y_j' is feasible for all j ($y_j' \in Y_j$, all j)

But this can only be true if at least one firm j was <u>not</u> maximizing profits at prices p in the original allocation. This contradicts the definition of WE.

Therefore, (x, y) must be PE.

<u>Second Theorem:</u> The separating hyperplane method generalizes to the case with production fairly easily. It is still necessary to redistribute endowments (now including the T_{ij} shares)

The cheap short cut where we <u>assume</u> existence of an equilibrium doesn't work so well though

(it involves setting $T_{ij} = 0$ for all i, j, so we would be throwing away some profit in firms where profit is positive).

The Robinson Crusoe economy

Suppose there is just one consumer and one firm (price-taking behavior doesn't make much sense here, but it is just an example).

Let's see how to represent the equilibrium graphically. Suppose the consumer (Crusoe) is endowed with T units of time; let H = work hours, L = leisure, $H + L = T$.

Crusoe has the utility function $u(c, L)$ where c = consumption, L = leisure. Assume no endowment of consumption (think of it as food).

Crusoe solves

$$\max u(c, L) \text{ subject to } pc = w(T-L) + \pi(p, w)$$

$$\text{or } \quad pc + wL = wT + \pi(p, w)$$

where p = price of food

w = wage

$\pi(p, w)$ = profit of firm

Note that Crusoe gets 100% of the profit from the firm.

Also note that we can write the budget constraint as

$$c = \frac{wT + \pi(p, w)}{p} - \frac{wL}{p} \qquad \left(\text{slope} = -\frac{w}{p}\right)$$

The firm maxes profit, so it solves

$$\pi(p, w) \equiv \max \; pf(H) - wH$$

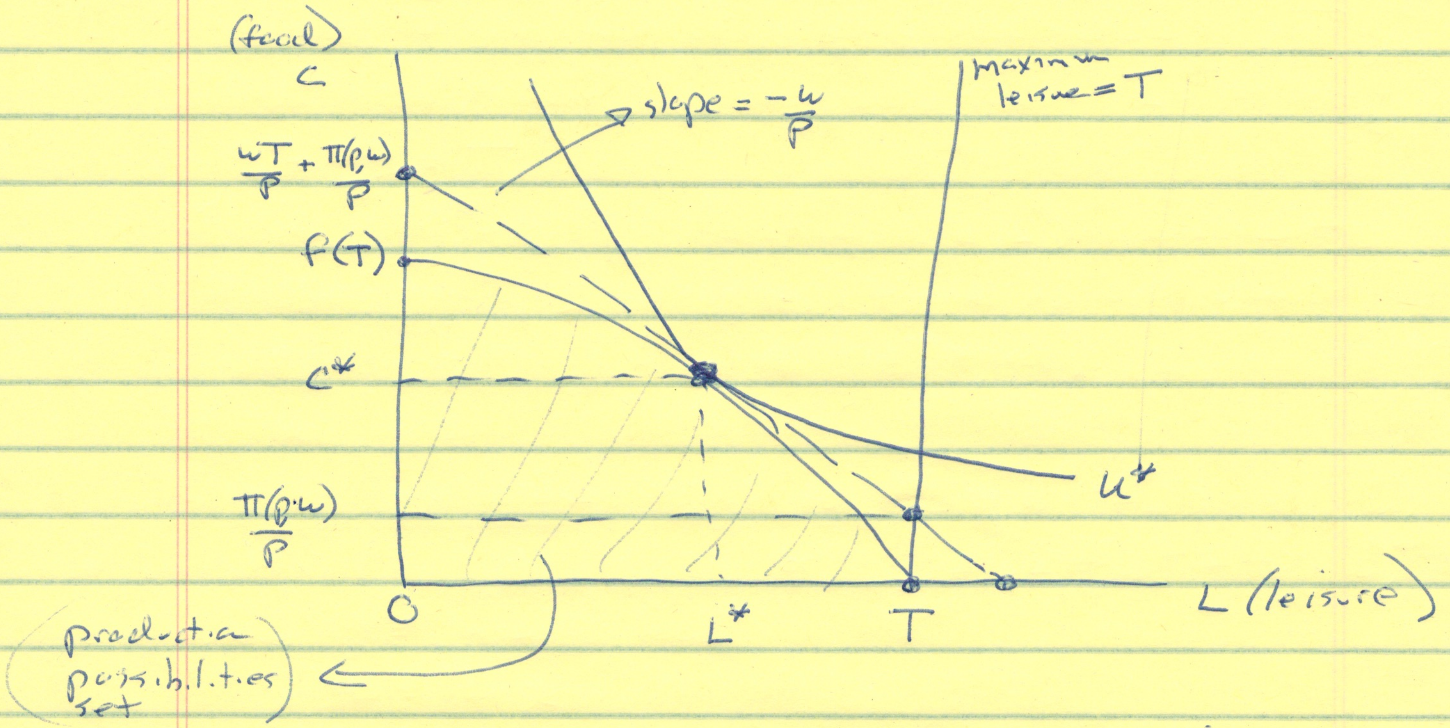
where $c = f(H)$ is the amount of food produced from H hours of labor

(assume decreasing returns so this has a solution)

We can think of the firm as having isoprofit lines of the form $pc - wH = \pi$ or $c = \dfrac{\pi + wH}{p}$ where π is constant along an isoprofit line.

The firm wants to get to the highest possible isoprofit line.

Here is a graph that shows the situation:



If we have no prices and just want Pareto efficiency, we max utility subject to technology and resource constraints.

Note that $L = T$ would imply $H = 0$, so $f(0) = 0$ and $c = 0$.

On the other hand, $L = 0$ implies $H = T$, so $c = f(T)$, which gives the vertical intercept.

The problem reduces to max $u(c, L)$ subject to $c = f(T - L)$ and $0 \le L \le T$ which gives an optimal point like (L^*, c^*) with utility u^*.

Notice that the solution occurs at a tangency point where the slope of the indiff curve = slope of production function.

Now let's see how we can use prices to construct a Walrasian equilibrium (Second Theorem of Welfare Economics).

Let p = price of food, w = price of labor.

Construct a budget line $c = \dfrac{wT + \pi(p,w)}{p} - \dfrac{wL}{p}$
as on p. 9. The vertical intercept
has $L = 0$ and $c = \dfrac{wT}{p} + \dfrac{\pi(p,w)}{p}$ (see graph)
The horizontal intercept has $c = 0$ and
$L = T + \dfrac{\pi(p,w)}{w}$. But because $\pi(p,w) > 0$,
we have $L > T$, which is not really feasible
(see graph). The more relevant case is $L = T$,
which gives $c = \dfrac{\pi(p,w)}{p}$; you can interpret this
as the amount of food Crusoe can buy using
his non-labor income (again, see graph)

Given this budget line, Crusoe can afford (L^*, c^*) and
no other feasible point is better, so he is max'ing utility

Now think about the firm. Its isoprofit lines have the form
$$c = \frac{\pi}{p} + \frac{w}{p}H = \frac{\pi}{p} + \frac{w}{p}(T-L) = \frac{\pi + wT}{p} - \frac{w}{p}L$$
so in (L,c) space the slope is $-\frac{w}{p}$, which is the
same slope as Crusoe's budget line. Thinking of the
shaded area as the production possibilities set,
the firm wants to get to the highest possible
isoprofit line, which occurs at (L^*, c^*).

Finally, think about market clearing.

In the food market Crusoe is demanding c^* at the prices (p, w) and the firm is supplying $c^* = f(T - L^*)$ so supply = demand for food.

In the "leisure market" Crusoe is demanding L^*, and sells $T - L^* = H^*$ hours of labor time to the firm. This is the quantity of labor the firm is demanding at the prices (p, w), so supply = demand for labor.

This shows that we can construct a Walrasian equilibrium graphically for an economy with production (although it is a very simple economy!)

We have found prices such that

1. the consumer is maxing utility subject to the budget constraint
2. the firm is maxing profit subject to a technology constraint
3. supply = demand for all goods,

The graph also gives some intuition about why the welfare theorems hold:

1. a WE is PE (the equilibrium allocation maxes utility subject to technology and resources)
2. a PE allocation can be supported by a WE (we can find prices that lead to a WE where the resulting allocation is the one we wanted).